	P.R.Government College (Autonomous) KAKINADA	Program&Semester II B.Sc. Major & Minor (IV Sem) w.e.f 2023-24 admitted batch			
Course Code MAT- 402 T	**TITLE OF THE COURSE** **Introduction to Real Analysis & Problem Solving Sessions**				
Teaching	HoursAllocated:60(**Theory**)	L	T	P	C
Pre-requisites:	Knowledge of Multivariate Calculus and Linear Algebra	3	1	-	3

Course Objectives:

To formalise the study of numbers and functions and to investigate important concepts such as limits and continuity. These concepts underpin calculus and its applications.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Get clear idea about the real numbers and real valued functions.
CO2	Obtaintheskillsofanalysingtheconceptsandapplyingappropriatemethodsfortesting
CO3	Test the continuity and differentiability and Riemann integration of a function.
CO4	Know the geometrical interpretation of mean value theorems and the fundamental theorem of integral calculus

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development		Employability		Entrepreneurship	

Unit – I

REALNUMBERS,REAL SEQUENCES

The algebraic and order properties of R - Absolute value and Real line - Completeness property of R -Applications of supremum property - intervals. (No question is to be set from this portion) Sequences and their limits -Range and Boundedness of Sequences - Limit of a sequence and Convergent sequence -The Cauchy's criterion - properly divergent sequences - Monotone sequences -Necessary and Sufficient condition for Convergence of Monotone Sequence - Limit Point of Sequence -Sub sequences and the Bolzano-weierstrass theorem – Cauchy Sequences – Cauchy's general principle of convergence.

Unit – II

INFINITIE SERIES

Introduction to series -convergence of series -Cauchy's general principle of convergence for series tests or convergence of series - Series of non-negative terms - P- test - Cauchy's nth root test -D'-Alembert's Test.

Unit – III

LIMIT & CONTINUITY

Real valued Functions - Boundedness of a function - Limits of functions - Some extensions of the limit concept - Infinite Limits - Limits at infinity (No question is to be set from this portion).Continuous functions - Combinations of continuous functions - Continuous Functions on intervals - uniform continuity.

Unit IV:

DIFFERENTIATION ANDMEANVALUETHEORMS

The derivability of a function at a point and and on an interval - Derivability and continuity of a function -Mean value Theorems -Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem.

Unit – V

RIEMANNINTEGRATION

Riemann Integral - Riemann integral functions - Darboux theorem -Necessary and sufficient condition for R integrability - Properties of integrable functions - Fundamental theorem of integral calculus - Mean value Theorems.

Activities

Seminar/ Quiz/ Assignments/ Applications of ring theory concepts to Real life Problem /Problem Solving Sessions.

Text book

Modern Algebra by A.R.Vasishta and A.K.Vasishta, Krishna Prakashan Media Pvt. Ltd.

Reference books

1. A First Course in Abstract Algebra by John. B. Farleigh, Narosa Publishing House.
2. Linear Algebra by Stephen. H. Friedberg and Others,Pearson Education India

CO-POMapping:

(1:Slight[Low]; 2:Moderate[Medium]; 3:Substantial[High], '-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PSO1	PSO2	PSO3
CO1	3	3	2	3	2	3	1	2	2	3	2	3	2
CO2	3	2	3	3	2	3	3	1	3	2	3	2	1
CO3	2	3	2	3	2	3	2	2	2	3	2	2	3
CO4	3	2	3	2	2	1	3	3	2	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-IV

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	REAL NUMBERS, REAL SEQUENCES	1	1	15
II	INFINITIE SERIES	2	2	30
III	LIMITS & CONTINUITY	2	1	20
IV	DIFFERENTIATION AND MEAN VALUE THEORMS	1	1	15
V	RIEMANNINTEGRATION	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)
E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M
Essay questions : 3 X 10 = 30 M
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Total Marks = 50 M
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Pithapur Rajah's Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - IV Semester
Mathematics Course X: Introduction to Real Analysis
Model Paper (w.e.f. 2024-25)

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Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer any three questions selecting atleast one question from each part

Part – A **3 X 10 = 30 M**

1. Essay question from unit - I.
2. Essay question from unit - II.
3. Essay question from unit - II.

Part – B

4. Essay question from unit - III.
5. Essay question from unit - IV.
6. Essay question from unit - V.

SECTION-B

Answer any four questions **4 X 5 M = 20 M**

7. Short answer question from unit – I.
8. Short answer question from unit - II.
9. Short answer question from unit – II.
10. Short answer question from unit - III .
11. Short answer question from unit – III.
12. Short answer question from unit – IV.
13. Short answer question from unit - V

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–IV: REAL ANALYSIS

Short Answer Questions

UNIT-I

1. Show that every convergent sequence is bounded . Give an example to show the converse is not true .

2. Prove that $\lim \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2}\right] = 0$

3. Show that $\lim_{n\to\infty} \left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \cdots + \sqrt{\frac{1}{n^2+n}}\right] = 1.$

4. State and prove sandwich theorem .

5. Define a Cauchy's sequence . Prove that every convergent sequence is a Cauchy sequence

6. Prove that if $\{S_n\}$ is a Cauchy sequence, then $\{S_n\}$ is bounded.

7. Prove that every Cauchy's sequence is convergent.

UNIT-II

8. If $\sum u_n$ convergence then show that lim u_n = 0 . Is the converse true ? Justify your answer .

9. Test for convergence of $\sum \frac{1}{2^n+3^n}$

10. Test for convergence of $\sum (1+\frac{1}{n})^{-n}$

11. Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3}\right)$.

12. Test for convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3-1}\right)$

13. Test the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^3}\left(\frac{n+2}{n+3}\right)^n x^n,\ \forall\, x > 0$

14. Test for convergence of $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

15. Test for the convergence of $\sum_{n=1}^{\infty} \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n} x^{n-1}\ (x > 0)$.

UNIT-III

16. If f: [a, b] → R is continuous on [a, b] then f is bounded on [a, b].

17. Examine the continuity of the function defined by f(x) = $|x| + |x-1|$ at x = 0, 1.

18. Show that $f: R \to R$ defined by f(x) = 1 if x ∈ Q ; f(x) = -1 if x ∈ $R - Q$ is discontinuous for all x ∈ R .

19. Discuss the continuity of the following function at the origin $f(x) = x\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ $if\ x \neq 0$ and f(0)=1 .
20. Prove that the function defined by f(x) = x^2 sin(1/x) for x ≠ 0 and f(x) = 0 for x = 0 is continuous at x = 0.
21. Discuss the continuity of $f(x) = x\left(\frac{e^{1/x}}{e^{1/x}+1}\right)$ $if\ x \neq 0$ and f(0) = 0 at the origin.
22. From the definition prove that f(x) = x^2 is uniformly continuous on [-a, a].
23. From the definition show that f(x) = x^2 + 3x is uniformly continuous on [-1, 1]

UNIT-IV

24. If $f: [a,b] \to R$ is derivable at $c \in [a,b]$, then prove that f is continuous on $[a,b]$. Is its converse true? Justify your answer.
25. Find c of Cauchy's mean value theorem for $f(x) = \sqrt{x}, g(x) = \frac{1}{\sqrt{x}}$ in [a, b] where $0 < a < b$.
26. Verify Cauchy's mean value theorem for f(x) = x^2 , g(x) = x^3 in [1,2] .
27. Examine the applicability of Rolle's theorem for $f(x) = 1 - (x-1)^{2/3}$ on [0, 2].
28. Discuss the applicability of Rolle's theorem for the function $f(x) = log\frac{x^2+ab}{x(a+b)}$ in [a,b] where $0 \notin [a,b]$.
29. Using Lagrange's Mean Value theorem prove that $1+x < e^x < 1+xe^x$, $\forall x > 0$

UNIT-V

30. If $f(x) = x^2\ \forall\ x \in [0,1]$ and P= {0,1/4 , 2/4, 3/4, 1}, then find $U(P,f)$ and $L(P,f)$.
31. Find the lower and upper Riemann sums of f(x) = 2x -1 on [0,1] when P= { 0, 1/3 , 2/3 , 1 } .
32. If $f \in R[a,b]$ and m , M are the infimum and supremum of f on [a,b] then prove that $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$.
33. If f is R-integrable on [a,b] then show that | f | is R-integrable on [a,b] .
34. Evaluate $\int_0^{\pi/4}(sec^4 x - tan^4 x)dx$

Essay Questions

UNIT-I

1. Show that a monotonic sequence is convergent iff it is bounded .
2. Prove that the sequence $\{s_n\}$ defined by $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ is convergent.

3. Show that the sequence $\{s_n\}$ where $s_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$ is convergent .

4.If $S_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)}$ the show that $\{s_n\}$ is convergent .

5.State and prove Cauchy's general principle of convergence .

UNIT –II

6.State and prove Limit comparison test.

7.State and prove Cauchy's n^{th} root test .

8.State and prove D'Alemberts ratio test.

9.Test for convergence $\sum \frac{x^n}{x^n+a^n}$ (x>0, a>0).

10.Test for convergence of i) $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1} - n\right)$ ii) $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1} - \sqrt{n^4-1}\right)$.

11. Test for convergence of $\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \cdots$

UNIT-III

12. State and Prove Intermediate value theorem.

13.If $f: [a, b] \to R$ is continuous on $[a, b]$, then f is bounded on $[a, b]$ and attains its bounds or infimum and supremum.

14.Test the continuity of of $f(x) = \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\ if\ x \neq 0\ and\ f(0) = 0$ at $x = 0$.

15.Let f: R→ R be such that f(x) = $\frac{sin(a+1)x + sinx}{x}$ for $x < 0, f(x) = c\ for\ x\ = 0$ and

$f(x) = \frac{(x+bx^2) - x^{\frac{1}{2}}}{bx^{\frac{3}{2}}}$ for x >0. Determine the values of a, b, c which the function is continuous at x = 0.

16. Determine the constants a,b so that the function defined by f(x) = 2x + 1 if x ≤ 1 ;
f(x) = ax^2 + b if 1< x < 3 ; f(x) = 5x + 2a if x ≥ 3 is continuous every where .

UNIT-IV

17. State and prove Roll's theorem .

18. State and Prove Lagrange's mean value theorem .

19. State and Prove Cauchy's mean value theorem.

20. Using Lagrange's mean value theorem, show that

$$x > log(1+x) > \frac{x}{1+x}\ if\ f(x) = log(1+x)\ \forall x > 0$$

21. Show that $\frac{v-u}{1+v^2} < tan^{-1}v - tan^{-1}u < \frac{v-u}{1+u^2}\ for\ 0 < u < v$. Hence deduce that

$\frac{\pi}{4} + \frac{3}{25} < tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

UNIT-V

22. State and prove Necessary and Sufficient condition for integrability.

23. If f is continuous on the $[a, b]$ then prove that f is Riemann Integrable on the $[a, b]$.

24. If f is monotonic on the $[a, b]$ then prove that f is Riemann Integrable on the $[a, b]$.

25. State and prove fundamental theorem of Integral Calculus.

26. Prove that $\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3\,cosx}\,dx \leq \frac{\pi^3}{6}$.

27. Show that $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2}\,dx \leq \frac{2}{\pi}$

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